OUTLINE FOR MTH1022 PRESENTATION ON *The Tantalizing Four Cubes*

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1. Introduction

In this presentation, we will apply graph theoretic solutions to a notable combinatorial problem involving a set of four cubes with colored faces. The problem was condensed into a game and sold under a few different names in North American market in the past century. Most popularly dubbed “Instant Insanity” or “The Great Tantalizer”, there exist varying versions of the game, however most follow suit of the original configuration, which allowed for only one solution. Some other versions, although, contain their own unique color configurations and number of solutions to the general problem.

1. Summary of the problem
2. Given the problem includes 4 separate cubes and 4 unique colors, the total number of possible arrangements is **41,472**. This number is calculated as follows:

**(1)** There are 3 ways in which the first cube may be chosen, by first deciding which pair of faces will be left-right pair. Only 3 pairs exist on a cube, therefore the first choice is simplest.

**(2)** This number is then multiplied by the number of ways in which a following cube may be added to the subsequent in relation to it. In this case, there are 24 ways in which a new cube may be added in relation to an established sequence of cubes. This is then repeated for each remaining cube to be added to the sequence, finally generating a total of **3 x 24 x 24 x 24 = 41,472.**

1. Given the fact that there is a single solution for a configuration of cubes, the probability of generating a solution in a completely random way would be 1 in 41,472, or roughly a 0.002% chance. This is more than three times less likely than the chance you will be struck by lightning in your lifetime, which is quoted as about a 1 in 13,000 chance (as per the National Oceanic and Atmospheric Administration: http://www.lightningsafety.noaa.gov/odds.shtml).
2. Instant Insanity puzzle and solution description\*\*
3. Overview of graph theory:

Graph theory is a discipline within mathematics that handles the study of mathematical structures known as ***graphs***. A graph is defined as a series of related nodes, or ***vertices***, which represent entities or states within a given mathematical system. Relationships between vertices of a graph are represented by ***edges***, which are illustrated in a graph using lines or curves between a set of vertices. An edge which connects a vertex to itself is called a ***loop***. A vertex of a graph is said to have ***nth degree*** when said vertex contains n-number of edges connected to it.   
 The implied goal of employing a graph-theoretic approach to a given problem is to represent a set of data, and any relationships inherent within that set, in a more intuitive way, and to create a model of a problem in such a way that one may find more profound understanding of the features and characteristics governing the problem.

1. Our own examples:

* A case that does not work & why
* A case that works & why, how to solve it

1. Applications of graph theory:

* Classic applications: travelling sales man, the four color theorem
* Practical applications: genomics, communication networks (e.g. circuit design, webpage links), scheduling (e.g. traffic planning, flights, etc.), other areas of pure mathematics, quantum computing, crowd control, disease spread
* \*\* Google’s solution to web search good pages have many links (edges), while less good pages are quite “lonely” in the Internet graph.

(References:

* 1. <http://world.mathigon.org/Graph_Theory>

1. A computer program:

* A program that takes as input the pattern of colors on a set of four colored cubes and finds all solutions (if any exist)