**A Tantalizing Twist with Cubes**

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1. Introduction

In this presentation, we will apply graph theoretic solutions to a notable combinatorial problem involving a set of four cubes with colored faces. The problem was condensed into a game and sold under a few different names in North American market in the past century, the most popular names being “Instant Insanity” and “The Great Tantalizer.” There exist varying versions of the game, but most follow suit of the original configuration, which allowed for only one solution. The general problem, however, allows any configuration of colors, and thus can have more than one solutions, or even none. We will briefly discuss graph theory, its applications, and how we used it to implement a supplemental computer program to solve the puzzle given any specific color input.

1. Summary of the problem

The general problem can have more (or fewer) cubes, faces, and colors, but we will limit ourselves to four-faced cubes with four colors. We call it the Four Cubes Problem for ease of address.

1. The goal of the Four Cubes Problem is to arrange four separate cubes, each colored one of four colors on each face, in a way such to create a “log” of cubes, which is one cube, one cube deep, and four cubes wide. The log of cubes is held together, and rotated as a whole to show a combination of the faces of each cubes, creating a multicolored “log-face” at each rotation.

**(possible graphics here)**

1. The combined sequence of the colored faces must not repeat any color on the same face of the log, but must also have each color appear once on each face of the log. Given the problem includes four separate cubes and four unique colors, the total number of possible arrangements is **41,472**. This number is calculated as follows:

**(1)** The first cube are chosen by deciding which pair of faces will be left-right pair. Only three pairs exist on a cube, therefore there are three ways to arrange the first cube.

**(2)** This number is then multiplied by the number of ways in which a following cube may be added to the subsequent in relation to it. In this case, there are 24 ways in which a new cube may be added in relation to an established sequence of cubes. This is then repeated for each remaining cube to be added to the sequence, finally generating a total of **3 x 24 x 24 x 24 = 41,472** for four cubes.

1. Given the fact that there is a single solution for a configuration of cubes, the probability of generating a solution in a completely random manner would be 1 in 41,472, or roughly a **0.002%** chance. This means one is three times less likely to generate a solution randomly than one is to be struck by lightning in one’s lifetime, which is quoted as about a 1 in 13,000 chance [1].
2. Overview of graph theory

Graph theory is a discipline within mathematics that handles the study of mathematical structures known as ***graphs***. A graph is defined as a series of related nodes, or ***vertices***, which represent entities or states within a given mathematical system. Relationships between vertices of a graph are represented by ***edges***, which are illustrated in a graph using lines or curves between a set of vertices. An edge which connects a vertex to itself is called a ***loop***. A vertex of a graph is said to have ***nth degree*** when said vertex contains n-number of edges connected to it.   
 The implied goal of employing a graph-theoretic approach to a given problem is to represent a set of data, and any relationships inherent within that set, in a more intuitive way. This allows one to create a model of a problem, through which one may hope to find a more profound understanding of the features and characteristics governing the problem.

1. How does Graph Theory help solve this puzzle?
   1. Reasoning:

**(4 crosses here)**

The edges in a graph help visualize the relationships between opposite-side pairs, which are important because the color of one face forces that of the opposite face. Faces are considered in opposite-side pairs because it is dimensionally impossible to rotate a cube in such a way that one face moves and its corresponding opposite face does not. In contrast, we can move a given face and not necessarily move its neighboring or adjacent faces.

(one of the cubes are displayed here, in 2 states: before move, after move)

Since one side of a log needs one of each color, two opposite sides of the log will aggregately have two of each color. To achieve this goal, a possible graph representation can have vertices as colors and their degree as how many times they appear on a log. This leads us to the following choice of graph construction, where edges represent pairs and vertices represent colors, making it easy to measure the degree of each color and visualize the connections of pairs.

Before the algorithm, some definitions need to be introduced:

* **Master graph**: …..
* A **valid subgraph** is a …..
  1. Algorithm:
* Create a master graph
* Extract valid subgraphs. A valid subgraph is one that …
* …

(a master graph. Caption: a master graph’s definition)

* 1. (Our own) examples: (**can put on board)**
* A case that does not work & why
* A case that works & why, how to solve it

1. Other applications of graph theory: (**graphics, put on board)**

* Classic applications: travelling sales man, the four color theorem
* Practical applications: genomics, communication networks (e.g. circuit design, webpage links), scheduling (e.g. traffic planning, flights, etc.), other areas of pure mathematics, quantum computing, crowd control, disease spread
* \*\* Google’s solution to web search good pages have many links (edges), while less good pages are quite “lonely” in the Internet graph.

(References:

* 1. <http://world.mathigon.org/Graph_Theory>

1. A computer program: (**might put on separate sheet of paper)**

We created a Java program which determines the solution(s) for a given set of cubes, if existent, and displays the solution. The program allows a user to decide the colors for each face of the four cubes. For a simplified program and as a means to better represent the principles behind the problem, the faces are broken up into opposite-side pairs. The program then takes the user input and works a graph theoretic approach to solve for the solutions. Firstly, the program creates the Cartesian product of all the pairs of each cube, creating all possible subgraphs of the cubes entered. It then runs a test and retains only the subgraphs that are considered a valid component of a solution. For every subgraph that passes this test, the program attempts to pair it, using another test, with another valid subgraph to create a valid final solution. The program will then output all possible solutions given the input data, and show how the inputted cubes must be oriented in regards to one another to achieve the physical solution.

1. References:
2. The National Oceanic and Atmospheric Administration: <http://www.lightningsafety.noaa.gov/odds.shtml>.
3. William C. Arlinghaus. The tantalizing four cubes. In John G. Michaels and Kenneth H. Rosen., editors, Applications of Discrete Mathematics, chapter 16. McGraw-Hill Higher Education, 1991.